

Mapping Problems of Geophysical Fields in Ocean and Extremum Problems of Underwater Objects Navigation

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Abstract: The paper focuses on mathematical and technical problems of navigation and control of unmanned underwater vehicles (AUV). Such problems arise during geophysical fields mapping, development of map-aided navigation methods, and optimal routes finding. To address map creation problem, the results of measurements on trajectory were used; then the field map reconstruction in fixed frame of a reference followed. The paper discusses optimization of the survey routes and optimization of measurements of the parameters of local fields and their anomalies on trajectory of an AUV. The field map reconstruction methods based on measurement results on trajectory are considered. Some mathematical problems of navigation by geophysical field maps are discussed. More specifically, matching algorithms for fragment taken in motion with a reference map are studied, the problem of the best reference map approximation for data compression on the AUV board is considered, the field informativity criterion is offered, and the problem of optimal route construction on the basis of such criterion is solved. The estimates obtained in the paper are based on theoretical studies, the results of simulation experiments, and field experimental trials of the AUV systems.

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1. INTRODUCTION

The paper relates to the multidisciplinary field being at the intersection of underwater robotics, oceanography, and mathematical theory of moving controlled objects. The paper deals with the actual problems of navigation and control of autonomous underwater vehicles (AUV) designed, in particular, for tracking survey of ocean geophysical fields. The problems of survey and mapping of local bathymetric, magnetometric, and gravitational fields and their anomalies are of the great practical interests. The overall goal of motion routes optimization is divided into several subproblems. They involve arrangement of search motions, map reconstruction based on the results of measurements along trajectory, and profiling the anomaly area by a closed trajectory. The first part of the paper deals with these problems.

Geodesic measurements with the help of aerospace, ground, and maritime facilities are nowadays widely used for the purposes of moving objects precision navigation as well as for the Earth geological structure survey. A special attention is paid to detailed study of anomalous fields in different regions of earth surface and seawaters, since they are of geodesic interest. Ocean physical fields survey and mapping are among classical oceanography goals. The problems of field characteristics measurement with precise

navigation data and with 3D imaging of survey results have special significance. In particular, the bathymetry, magnetometry, and gravimetry of different marine areas are of great interest. The works Ageev (1994), Kinsey et al. (2009), Inzartsev et al. (2010), Kiselev et al. (UT-2017), Kiselev et al. (ICINS-2017), and Inzartsev et al. (2018) are devoted to potential of AUV application to geodesic measurements at sea.

The second part of the paper analyzes problems connected with peculiarities of navigation by geophysical field (see Stepanov et al. (2015), Stepanov et al. (2016), Berdyshev et al. (2007)), including the following problems:

- the search on the reference map of the field fragment taken in the course of motion; such problem is called the problem of the *fragment matching* with the reference map of field;
- the best reference map approximation from the navigation precision point of view for effective storage of reference map on board of the underwater vehicle;
- the field informativity estimation and the most “informative” route construction, which means finding the best route from the point of view of navigation accuracy by the geophysical field when following such route.

2. GEOPHYSICAL FIELD ROUTING AND MAPPING PROBLEMS

Let $F(x, y)$ be a continuously differentiable function on $Q \subset \mathbb{R}^2$ with the axes (x, y) characterizing horizontal section of spatial geophysical field. The function $F(x, y)$ may be represented, for example, as an isocurve map $F(x, y) = \text{const}$. Field measuring device during motion along the trajectory $\{x(t), y(t)\}$, where t is the current time, gives the field measurement $\tilde{f}(t)$ with a random error $\xi(t)$

$$\tilde{f}(t) = f(t) + \xi(t), \quad f(t) = F(x(t), y(t)). \quad (1)$$

Let suppose that coordinates (x, y) and velocity vector projection (v_x, v_y) of the vehicle (measuring device) are defined by the on-board navigation system with errors $(\Delta x_a, \Delta y_a), (\Delta v_x, \Delta v_y)$. So, coordinates of a point $(\tilde{x}(t), \tilde{y}(t))$, to which the field measurement $f(t)$ at the instant t refers, differ from the true coordinates $(x(t), y(t))$ as follows:

$$\begin{aligned} \tilde{x}(t) &= x(t) + \Delta x_a + \Delta v_x t, \\ \tilde{y}(t) &= y(t) + \Delta y_a + \Delta v_y t. \end{aligned} \quad (2)$$

Define the field variability along the trajectory by the field gradient value $|\nabla F|$ or change $\Delta F = |\nabla F| |v| \Delta t$ in the time interval Δt .

The following interrelated problems are of a particular interest.

Problem 1 involves the field F map reconstruction in the form of intensity matrix on a regular grid or a set of isocurves. The field map should be reconstructed by covering a specified area $Q \subset \mathbb{R}^2$ with trajectories grid and measuring (1) the field parameters with reference to points (2) obtained on the basis of inaccurate navigation data.

As a rule, underwater vehicle control comes down to creation and correction of the course program at distinguished points (x_k, y_k) of the trajectory, which may be determined by the conditions of field local extreme values or by the condition of field gradient maximum module.

Problem 2 involves search of anomalies manifested in function F as an area of local extreme values. To perform such, search the underwater vehicle motions will be arranged such that to find and to profile the anomaly by fixed level $F(t)$ or by a fixed changeability $|\Delta F(t)|$ of signal of measuring device under the conditions of external disturbances. The search of coordinates with representative (in particular, extreme) values of field parameters is required to determine the landmarks for goal-seeking motions. Search procedure may be arranged on the basis of the orthogonal (fastest) or gradient descent of other type with orientation to the selected target, in this case, the source of anomaly. Accumulation of data on the field extreme values allows outlining anomaly or zone (background) where the anomalies are absent.

Problem 3 involves tracing the preset isocurve $F(x, y) = \text{const}$ determining anomaly boundary on the basis of preset (extreme) field level. Motion is defined by the velocity vector orientation according to the character of field change in proportion to the value $\nabla F_x / \nabla F_y$.

In general case, a scenario of search program mission includes the following processes:

- the arrival to the area with preset field level by the help of orthogonal tacks and course correction based on the field gradient sign change;
- the coverage of the found area with the grid of straight square meander-shaped trajectories (tacks) with field parameters measuring and their matching with navigation data for further field map reconstruction;
- the search motions by means of the orthogonal (fastest) descent to determine probable extremum value location;
- the tracking of given isocurve for determining anomaly boundary by means of variation of the velocity vector orientation in accordance with the changing “curvature” of field or motion by reckoned coordinates corresponding to the preset field level.

The main process scenario involves the field map reconstruction by the network of discrete measurements obtained by means of area coverage with the grid of straight trajectories (tacks). Results of measurement on the irregular grid are interpolated onto nodes of regular one. Interpolation pitch depends on the field gradient size at node points. Based on the data obtained, isocurves with the preset field level values are constructed.

A special (but in practical terms important) case is the control of motion in the anomaly area. When elaborating algorithms for AUV motion for anomalies survey, the following specific features of field spatial structure were taken into account. Those are: presence of natural background, temporal and spatial disturbances of the field, and essential non-linearity in their mathematical description.

The AUV mission-program noted above was investigated on a simulation model using MATLAB Simulink package with the following tools:

- DEE toolbox (differential equations system solver);
- StateFlow (event descriptions tools);
- RealTimeWorkShopEmbeddedCode (using the built-in model code).

Functional diagram of the simulation computer model, implemented by means of the package is shown in Fig. 1.

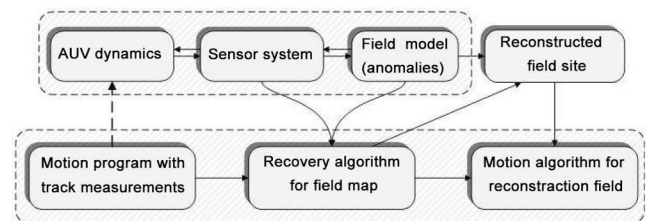


Fig. 1. Functional diagram of a simulation computation model

An add-on to the algorithm of orthogonal descent is the fuzzy logic one based on expert estimates of a specialist in a particular physical area. By the production rules of the fuzzy regulator, the expert's knowledge of the anomalous field is transformed into the target parameters of the search algorithm: the number of samples in the gradient stack, the target velocity of the AUV, the number of tacks before the search algorithm finishes, etc.

Figure 2 demonstrates the results of simulation experiment aimed at the local anomaly survey with the help of

autonomous underwater vehicle. The field structure is of an anomaly nature with extremum by level or gradient. To determine the extremum location, a search motion is carried out with arbitrary initial direction and trajectory changes at preset time interval. The value of field gradient in the current direction is determined by several samplings. In the case of gradient sign change, a turn to orthogonal tack takes place. Depending on field spatial changeability, the interval of sampling receipt is adjusted with the vehicle speed. When reaching anomaly area, the vehicle starts covering it with the grid of straight trajectories.

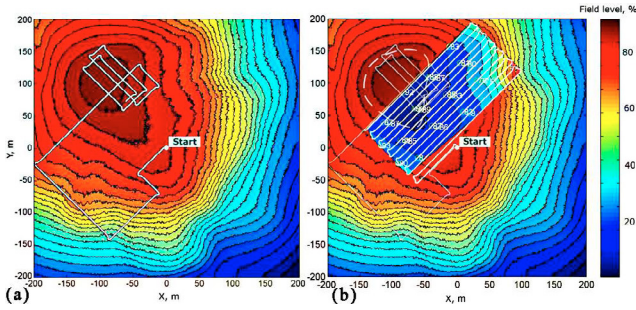


Fig. 2. Search trajectory (a) and field map fragment reconstruction (b)

The final search procedure involves profiling the area taken for the anomaly. A closed isocurvature with preset value of the reconstructed field level is taken as a boundary of such area.

Underline as a specific case, a reconstruction of magnetic anomaly field map tile formed by two closely-spaced magnetized bodies (Fig. 3). In the case of anomaly identification by detection system signals, one has to deal with complicated field implementations along the vehicle trajectory. As a rule, the signal that helps distinguishing magnetic anomaly from the “earth noise” is asymmetrical, and it possesses alternating polarity in the case of anomaly crossing in different directions at different distances. So, when constructing a system for finding the magnetic anomaly, it is necessary to distinguish a magnet contact against the background of broadband disturbance.

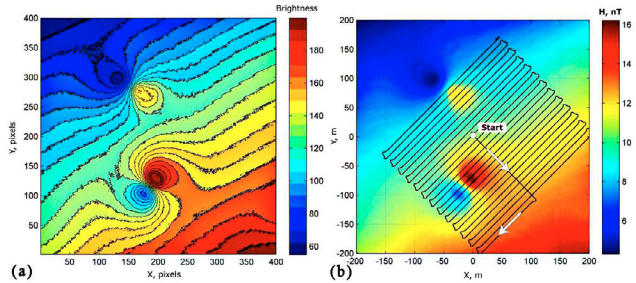


Fig. 3. Initial fragment of magnetic anomaly field (a) and its reconstruction based upon trajectory measurement results (b)

Obtaining the bathymetric measurements with control by *bottom configuration* (underwater relief) can be another real-world example of field trajectory survey with the help of AUV. Underwater relief may be represented as a field characterizing the sea depth $F(x, y)$ and, accordingly, the bathymetric contours have the form of $F(x, y) = \text{const}$.

Here, $F(x, y) = H(x, y) + h(x, y)$, where $H(x, y)$ is the vehicle operational depth, $h(x, y)$ is the above ground altitude. In general case, the problem is to construct a 3D-image of underwater relief and divide it into fragments for coverage by the grid of trajectories for the purpose of bathymetric map reconstruction. Scanning selected underwater relief fragment provides required information for reconstruction of 3D-model of relief with different view points and construction of relevant bathymetric map fragment. Then the problem is to analyze the image obtained, to distinguish peculiarities and arrange motion by the bathymetric contour, for example, to profile anomalous areas.

In general case, the use of data about field equals to inclusion of the field parameter $F(x, y)$ into extended vector of dynamic system condition with additional equation (see (1))

$$\dot{f} = \nabla F_x(x, y)|v| \cos \chi + \nabla F_y(x, y)|v| \sin \chi,$$

where χ is the angle of trajectory turn (angle between the x axis and velocity vector v).

During motion by isocurve $F(x, y) = F_0$ the velocity vector components (v_x, v_y) must “in the mean” obey (in reference to preset isocurve) to kinematic condition

$$\nabla F_x v_x + \nabla F_y v_y = 0.$$

This equation provides the values for programmed course if another motion parameters are known.

Figure 4 represents a simulation example of bathymetric contour motion that profiles the area with maximum depth in the water area. The bathymetric contour motion is arranged by means of setting a “tube”. Its width varies depending on the size of the field gradient. Since condition of rather large gradient and small “tube” width, the route points are evenly distributed along the bathymetric contour. Since “tube” width increase (gradient decrease), the route points are unevenly distributed and the bathymetric contour motion “in the mean” takes place.

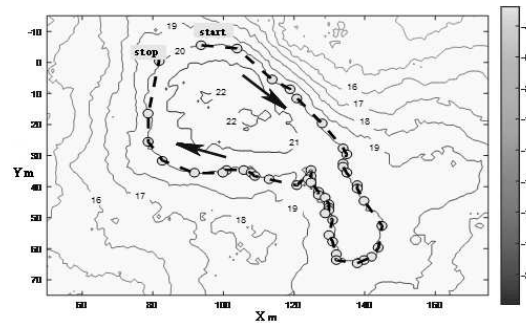


Fig. 4. Profiling of specific bottom configuration area in case of bathymetric contour motion

3. GRAVIMETRIC EXTREMUM PROBLEMS

One of the basic requirements to gravity measurements is that perturbing and inertial accelerations, to which the gravimeter base is exposed, should be minimum (see Ageev (1994), Kiselev et al. (ICINS-2017)). From this, the AUV as a well-stabilized platform possesses a number of advantages in comparison with other gravimeter carriers (see Ageev (1994), Inzartsev et al. (2018)). A geographic map

of residual gravity anomalies in the Peter the Great Gulf obtained with the help of a special high-precision survey is further used to make certain estimates. A map fragment with extinct volcano having the changeable gradient of the force gravity anomalies in the lower part is chosen. A field 50×40 km in size was surveyed in this area by meander-shaped trajectories.

Then obtained digitized data were used for field map reconstruction with the help of MATLAB program. The matrix transformation scripts and bivariate interpolation function on an uneven grid GRIDDATE were used. The function structure includes syntax and functional descriptions for input data packages transformation.

A 3D-fragment of reconstructed map area is represented in Fig. 5. The map reconstruction precision is determined mainly by measurements sampling interval and the reciprocal of current gradient values. Along the digitized trajectory, data processing the occasional errors of integrated positioning system was performed. Moreover, the measurement errors represented by normal distributions with preset mean-square parameters were taken into account.

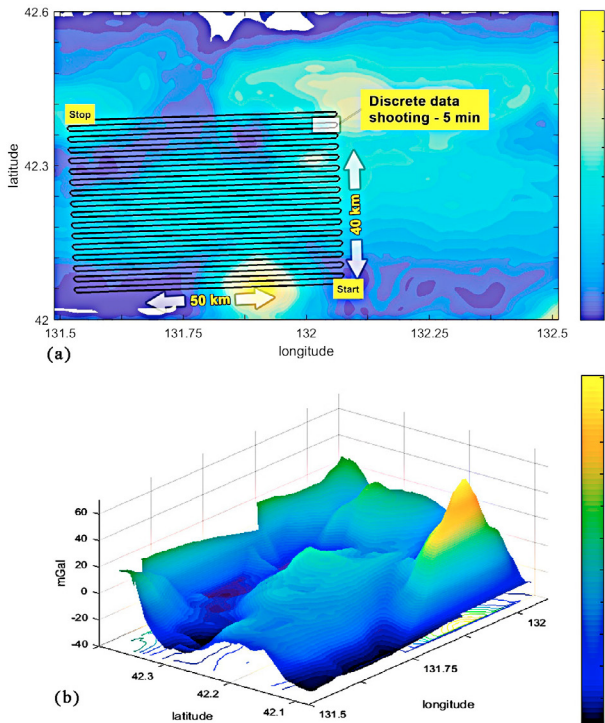


Fig. 5. Force gravity anomalies geographical map: initial fragment (a) and its reconstruction (b)

4. UNDERWATER GEOPHYSICAL FIELDS AS REFERENCE MAP FOR MAP-AIDED METHOD OF NAVIGATION

Underwater vehicle guidance system contains an inertial navigation subsystem (INS), which determines aboard the vehicle coordinates in some world coordinate system associated with the Earth. The operating principle of INS leads to the fact that the errors in determination of velocities and coordinates accumulate during the motion and grow with time.

One way to solve the error correction problem of the INS data is to use the map-aided method (see Stepanov et al. (2015), Stepanov et al. (2016), Berdyshev et al. (2007)). The working principle of the method is based on the comparison of measured fragment of the geophysical fields obtained during the vehicle motion (hereinafter called *the measured fragment*), with *the reference matrix*, which is stored aboard and calculated in advance. The matching of the measured fragment with the reference matrix performs error correction of INS. The matching is implemented by means of the *matching functional*. The estimate of location of the measured fragment in the coordinate system of the reference matrix is given by the argument of the extremal value of the matching functional.

In this section, the matching is performed by minimizing the quadratic norm of the squares sum of the differences in the values of the measured and reference fragments. The informativity of the geophysical field (from the point of view of map-aided method) is measured by the level of navigation errors when using this method. The sources of errors, as a rule, are the errors in cartography and ones in field measurements. However, the sensitivity of the result of navigation to these errors is mainly determined by the gradient characteristics of the field itself.

4.1 Statement of the problem of navigation by continuous field

Suppose that an underwater object (vehicle) has coordinates $q \in \mathbb{R}^3$ and moves over a region $Q \subset \mathbb{R}^2$ of the underwater medium and F is the geophysical field of the region (see Fig. 6). For example, $F(u) = (F_1(u), F_2(u))$ is a vector function, where $F_1(u)$ is the underwater depth of the seabed and $F_2(u)$ is either intensity of magnetic field or intensity of the field of gravity anomalies a point $u \in Q$. Information about the field on the whole is kept in the on-board computer of the object. Suppose that the object has a cone $\Delta = \{l\}$ of beams l of the multipath sonar. The object using the cone $\Delta = \{l\}$ from the position $w = (q, a)$ (where q is the center of mass of the object and a is an orientation vector) measures a fragment of the field

$$\varphi_w(l, F) = (\rho(q, f_{w,l}), F_2(u)), \quad l \in \Delta, \quad (3)$$

where u is projection of q into the plane (x, y) ,

$$f_{w,l} \in \text{graph } F_1, \quad \rho(q, f_{w,l}) = \rho(q, (a(l) + q) \cap \text{graph } F_1).$$

The problem of navigation (location) consists in finding the position $w(q, a)$ of the object by means of the fragment measured during the motion (see Fig. 6). This is the problem of inversion of the mapping $w \rightarrow \varphi_w$, which can be ill-posed.

The problem of navigation is reduced to the following problem:

$$d(w, F) = \min_{W \in \mathcal{W}} \|\varphi_w(l, F) - \varphi_W(l, F)\|_{\Delta},$$

where \mathcal{W} is the supposed region of search and $\|\cdot\|_{\Delta}$ is the prechosen norm on Δ .

To increase the accuracy of navigation, we can take the fragment from series of fragments of form (3) measured as follows. Let

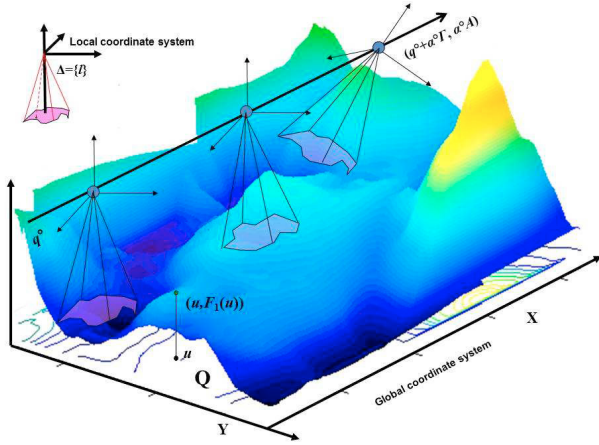


Fig. 6. Map-aided method of navigation: Geophysical field and measured fragment; here, the plane Q is considered to be located at sea level, but for convenience of drawing it is shown below the graph of the function $F_1(u)$

$$(\Gamma, A) = \{q(\tau) = (x, y, z), a(\tau) = (a_1, a_2, a_3), 0 \leq \tau \leq 1\} \subset \mathbb{R}^6, q(0) = 0, \Gamma = \{q(\tau)\}, A = \{a(\tau)\}$$

be a strapped-down fixed trajectory, for example, a segment of a straight line $a(\tau) = \text{const}$. The underwater object starts from an unknown position $w^0 = (q^0, a^0)$, implements the trajectory $(q^0 + a^0\Gamma(\tau), a^0A(\tau))$, and, at any instant $\tau_k = k/n$, $k = 1, \dots, n$, with chosen n , being on this trajectory, measures a fragment of form (3) for finding the initial position w^0 (see Fig. 6).

As an application of the proposed formulation of the geophysical field navigation problem, let us consider the navigation along the field of gravity anomalies. A fragment of the gravity map of the anomalous gravity field in the region of the Peter the Great Gulf is shown in Figs. 5 and 6. For the study, we chose the area with the coordinates $E131.5^\circ \dots E132.5^\circ, N42^\circ \dots N42.6^\circ$. As mentioned above, this is an area near the extinct volcano, located at a site with depths from 50 m to 3000 m, and, therefore, the gravity field gradient is very variable.

To obtain estimates with necessary accuracy of correction of navigation errors, we use the map-aided method of navigation by geophysical field. The method includes application of:

- the map of the gravity anomaly field described above having error level of cartography calculated in Kiselev et al. (ICINS-2017) of the order 2 – 5 mGal;
- the measurement model;
- the error model of measuring in the INS system.

Here, we consider the case of error correction by the position of the moving object.

The most accurate way to assess informativity is the method of statistical simulation of the process of correcting the navigation errors. This process involves the multiple formation of a sequence of measurements and their matching to the reference map. Such a simulation was carried out on the map depicted in Fig. 5(a) for the linear paths with length of about 60 km and directed

along the longitude axis. The points of measurements were located at 100 m intervals. Calculations were carried out for several zones of the region under consideration; in each zone up to 100 correction sessions were simulated. As a result of the experiments, we obtained the value $r = 476$ m for the mean radial error r of the position of the trace origin. This value was obtained under summary standard deviation $\sigma = 2$ mGal that is stipulated by errors of the field measurements and errors of the map reconstruction.

4.2 The best approximation of the geophysical field

In the case when the region Q is large, an information about F to be kept on the object board, probably, must be precompressed with the possibility of fast recovery. To this end, we consider a class $\mathcal{P} = \{p\}$ of functions p that can be given by a small number of parameters and are easily calculated. For specific $w \in \mathcal{W}$, a fragment $\varphi_w(l, F)$ ($l \in \Delta$) measured on the real geophysical field, and a function $p \in \mathcal{P}$, to solve the problem

$$d(w, p) = \min_{W \in \mathcal{W}} \|\varphi_w(l, F) - \varphi_W(l, p)\|_\Delta, \quad (4)$$

we will find a position $W(w, p)$ implementing the minimum and the navigation error $|w - W(w, p)|$, where $|\cdot|$ is the Euclidean norm in the six-dimensional space. For the functional

$$D(p) = \sup_{w \in \mathcal{W}} |w - W(w, p)| \quad (5)$$

given on the class \mathcal{P} , we will find the value

$$D = \min\{D(p) : p \in \mathcal{P}\} \quad (6)$$

and a function $p^* \in \mathcal{P}$ such that $D(p^*) = D$. The function $p^* \in \mathcal{P}$ provides the minimum of the maximum navigation error. We can approximate the geophysical field by a simpler method solving the classical problem on the best approximation

$$\min\{\|F - p\| : p \in \mathcal{P}\} = \|F - p^*\| \quad (p^* \in \mathcal{P}). \quad (7)$$

The function p^* is found by means of solution of the time-consuming problems (4)–(6). Methods for solving problem (7) in the case of linear classes \mathcal{P} are well known. However, computations show that, for a number of known geophysical fields, the navigation error $D(p^*)$ is 15–20% less than $D(p^{**})$ in the case of the mean-square norm $\|\cdot\|$ and the class \mathcal{P} of spline functions. Therefore, given the computational tools and time, it is reasonable to construct and apply the approximating function p^* .

4.3 Informativity of the geophysical field

A plain field yields little information, while the variable field is informative. We can characterize the informativity of the geophysical field F by means of the function (Berdyshev et al. (2007))

$$J(\tau, F) = \sup_{w, W \in \mathcal{W}} \left\{ |w - W| : \|\varphi_w(\cdot, F) - \varphi_W(\cdot, F)\|_\Delta \leq \tau \right\}, \quad \tau \geq 0, \quad (8)$$

which is called the *module of informativity*. To estimate the field informativity, the derivative $J'(\tau, F)|_{\tau=0}$ or the value

$$\int_0^\delta J(\tau, F) d\tau$$

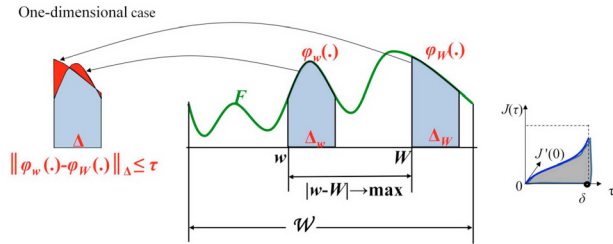


Fig. 7. Module of informativity

can be used for some $\delta > 0$, where $|\cdot|$ is the Euclidean norm (see Fig. 7). The less the function $J(\tau, F)$, the more informative the field is. Given the computational tools, we can precalculate the function $J(\tau, F)$ at a fine grid. The function $J(\tau, F)$ enables estimating the navigation error.

Theorem 1. The following inequality holds for $w \in \mathcal{W}$, $p \in \mathcal{P}$, and $W(w, p) \in \arg d(w, p)$:

$$|w - W(w, p)| \leq J(2\|F - p\|_Q, F).$$

5. OPTIMIZATION OF ROUTE FROM THE POINT OF VIEW OF INFORMATIVITY OF GEOPHYSICAL FIELD

In addition, the informativity module is useful when choosing a route over the most informative part of the region. For the trajectory

$$\mathcal{T} = \{q(\tau) : 0 \leq \tau \leq 1, q(0) = q_*, q(1) = q^*\} \quad (9)$$

connecting fixed points $q_* \neq q^*$ and its neighborhood $O_\varepsilon(\mathcal{T}) = \{q : \rho(q, \mathcal{T}) \leq \varepsilon\}$, $\varepsilon > 0$, we define the following function (similar to (8)):

$$J(\tau, F, \mathcal{T}, \varepsilon) = \sup_{q \in \mathcal{T}, T \in O_\varepsilon(\mathcal{T})} \left\{ |q - T| : \|\varphi_q(\cdot, F) - \varphi_T(\cdot, F)\|_\Delta \leq \tau \right\}.$$

Let Y be a connected domain containing $O_\varepsilon(\mathcal{T})$. It is required to find a trajectory $\mathcal{T}^* \subset Y$ of form (9) such that

$$\int_0^\delta J(\tau, F, \mathcal{T}^*, \varepsilon) d\tau = \min_{\mathcal{T} \subset Y} \int_0^\delta J(\tau, F, \mathcal{T}, \varepsilon) d\tau.$$

Figure 8 depicts a smooth trajectory \mathcal{T}^* presented by the graph of a spline-function obtained by the iterative method. The piecewise linear trajectory shown in Fig. 8 is taken as the starting one of the iterative process. It is seen that the resulting trajectory passes over the most broken part of the domain Y .

6. CONCLUSION

Our research has shown that underwater objects navigation problems are closely related to cartographic problems of the field of gravitational anomalies. This is primarily due to the difficulties in obtaining the field map, which leads to the need to develop a new navigation methods that take into account the specific features of the available information about geophysical field.

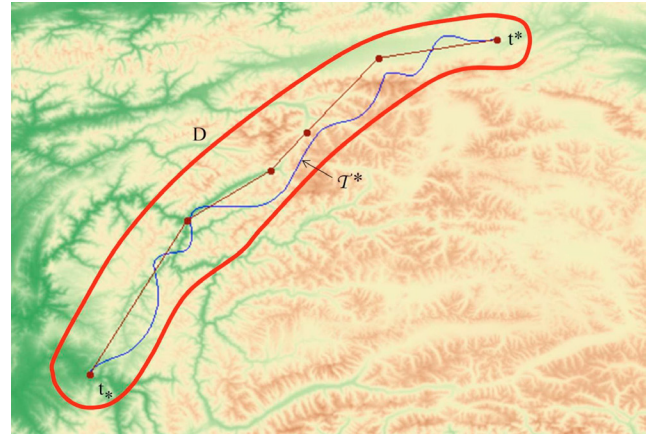


Fig. 8. Optimization of the route

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